ASYMPTOTIC STUDY OF A THREE-DIMENSIONAL VISCOUS SHOCK LAYER
IN THE NEIGHBORHOOD OF A PLANE OF SYMMETRY

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Despite the development of numerical methods for solving three-dimensional problems of the flow of a viscous gas about bluff bodies, it is still very important to continue to develop approximate methods of solving these problems that will yield simple analytic formulas. Such formulas, not requiring a large expenditure of machine time while providing sufficient accuracy, are widely used in performing engineering calculations. Many approximate methods have been developed for large Reynolds numbers Re, when flow is studied within the framework of boundary-layer theory. However, there are as yet no analogous methods suitable for solving three-dimensional problems of flow about bodies in the case of small Re, when viscosity is important over the entire perturbed region of flow.

The goal of the present investigation, being a continuation of [1], is to obtain simple formulas to determine the heat fluxes and shear stresses in the neighborhood of the plane of symmetry of bodies in a flow with an angle of attack. The investigation is conducted for small and moderate Reynolds numbers on the basis of approximate solution of the equations of a three-dimensional viscous shock layer with allowance for slip and the temperature jump on the surface. Problems in a similar formulation without allowance for slip were studied numerically in [2-5].

1. We will examine the three-dimensional flow of a viscous gas about bluff bodies at small and moderate Re, when the flow has a plane of symmetry. The flow is studied within the framework of a model which is analogous to the widely used two-layer model of a viscous shock layer proposed in [6] for axisymmetric flow about a body. The model is based on the assumption that the perturbed region of the flow is thin.

We choose a system of curvilinear orthogonal coordinates ($\xi^1$, $\xi^2$, $\xi^3$) which is normally connected with the surface in the flow: $\xi^3$ = const is the family of surfaces parallel to the surface of the body ($\xi^3 = 0$), while $\xi^1$ and $\xi^2$ were chosen on the surface in the following manner. Let $z = f(x, y)$ be the equation of the body's surface in a Cartesian coordinate system. The velocity vector of the incoming flow $\mathbf{V}_\infty$ coincides with the direction of the z axis. The coordinate origin is placed at the stagnation point of the flow. We introduce the following parametrization of the surface: $x = \xi^1$, $y = \xi^2$, $z = f(\xi^1, \xi^2)$. Let $\xi^2 = 0$ be the plane of symmetry of the flow. We expand all of the sought functions into series in $\xi^2$ in the neighborhood of this plane: $F(\xi^1, \xi^2, \xi^3) = F_0(\xi^1, \xi^3) + F_2(\xi^1, \xi^3)(\xi^2)^2 + \ldots$ We then insert these expansions into the equations of a three-dimensional thin viscous shock layer [7]. Keeping two terms in the expansion for pressure and one term in the expansions for the remaining functions, we obtain a closed system of equations to describe the flow in the neighborhood of the plane of symmetry. In variables of the Dorodnitsyn type, this system has the form

$$\beta_3 u^1 \frac{\partial u^1}{\partial \xi} - S\frac{\partial u^1}{\partial \zeta} + \beta_1 (u^1)^2 = -\frac{p_1}{\rho} + \frac{\partial}{\partial \zeta}\left(\frac{\mu\rho}{\mathrm{Re}\,\Delta^2}\frac{\partial u^1}{\partial \zeta}\right),$$

$$\beta_3 u^1 \frac{\partial u^2}{\partial \xi} - S\frac{\partial u^2}{\partial \zeta} + \beta_2 (u^2)^2 + \beta_4 (u^1)^2 + \beta_8 u^1 u^2 =$$

$$= -\frac{g}{\rho}(p_2 - \beta_0 p_1) + \frac{\partial}{\partial \zeta}\left(\frac{\mu\rho}{\mathrm{Re}\,\Delta^2}\frac{\partial u^2}{\partial \zeta}\right),$$

$$\beta_3 u^1 \frac{\partial G}{\partial \xi} - S\frac{\partial G}{\partial \zeta} = \frac{\partial}{\partial \zeta}\left[\frac{\mu\rho}{\mathrm{Re}\,\mathrm{Pr}\,\Delta^2}\frac{\partial}{\partial \zeta}\left(G - \frac{1-\mathrm{Pr}}{1-G_w}\beta_0 (u^1)^2\right)\right],$$

$$S = \beta_1\varphi^1 + \beta_2\varphi^2 + \beta_3\left(\frac{\partial \varphi^1}{\partial \xi} + \frac{\varphi^1}{\Delta}\frac{d\Delta}{d\xi}\right), \quad u^1 = \frac{\partial \varphi^1}{\partial \zeta}, \quad u^2 = \frac{\partial \varphi^2}{\partial \zeta},$$

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$$\sqrt{g}\frac{\partial p}{\partial \zeta} = \beta_4 \Delta (u^1)^2, \quad \sqrt{g}\frac{\partial p_1}{\partial \zeta} = g\beta_1\beta_3 u^1\left(u^1 \frac{d\Delta}{d\xi} + 2\Delta \frac{\partial u^1}{\partial \xi}\right) + \beta_5 \Delta (u^1)^2, \tag{1.1}$$

$$\sqrt{g}\frac{\partial p_2}{\partial \zeta} = \Delta\left[\beta_6 (u^1)^2 + 4\beta_7 u^1 u^2 + 2\beta_2^2 (u^2)^2\right],$$

$$\xi = \xi^1, \quad \zeta = \frac{1}{\Delta}\int_0^{\xi^3} \rho d\xi^3, \quad \Delta = \int_0^{\xi_s^3} \rho d\xi^3,$$

$$\frac{p}{\rho} = \varepsilon T, \quad T = G(1 - G_w) + G_w - \beta_0 (u^1)^2, \quad \mu = T^\omega,$$

$$G = \frac{H - H_w}{H_\infty - H_w}, \quad G_w = \frac{H_w}{H_\infty}, \quad \mathrm{Re} = \frac{R\rho_\infty V_\infty}{\mu(T_0)}, \quad T_0 = \frac{V_\infty^2}{2c_p},$$

$$\varepsilon = \frac{\gamma - 1}{2\gamma}, \quad g = 1 + f_1'^2, \quad p_1 = \frac{1}{g\beta_3}\frac{\partial p}{\partial \xi^1}\bigg|_{\xi^2=0}, \quad p_2 = \frac{1}{g\beta_2}\frac{\partial^2 p}{(\partial \xi^2)^2}\bigg|_{\xi^2=0},$$

$$\beta_1 = \frac{f_{11}''}{g^2}, \quad \beta_2 = \frac{f_{22}''}{g}, \quad \beta_3 = \frac{f_1'}{g}, \quad \beta_4 = f_1'^2\beta_1, \quad \beta_5 = \frac{f_{111}'''}{g^2} f_1' + \beta_1^2 g (6 - 4g),$$

$$\beta_6 = \frac{1}{\beta_2}\left[\frac{f_{1122}^{(IV)}}{g^2}\beta_0 + \beta_1\left(\beta_7 (6 - 4g) + 6\beta_2^2 (1 - g)\right)\right],$$

$$\beta_7 = \frac{f_{122}'''}{g^2} f_1', \quad \beta_8 = \beta_3 \frac{f_{122}'''}{f_{22}''} - 2\beta_4, \quad \beta_0 = \frac{f_1'^2}{g}.$$

Here, $\rho_\infty\rho$ is density; $\rho_\infty V_\infty^2 p$, pressure; $\mu\mu(T_0)$, viscosity; $HV_\infty^2/2$, total enthalpy; $TV_\infty^2/(2c_p)$, temperature; $V_\infty u_\infty^\alpha u^\alpha$, physical components of the velocity vector in the directions $\xi^1$ and $\xi^2$ ($V_\infty u_\infty^\alpha$ are the physical components of the velocity vector in the incoming flow); Pr, Prandtl number; $\gamma$, ratio of the heat capacities; R, characteristic linear dimension; the indices $\infty$, s, and w denote quantities in the undisturbed flow, behind the shock wave, and on the surface of the body; the indices 1 and 2 with the derivatives of the functions denote the coordinate ($\xi^1$ or $\xi^2$) with respect to which differentiation is performed.

On the surface of the body, we assign boundary conditions that consider slip velocity and the temperature jump [8]:

$$\zeta = 0\colon \; u^\alpha = Q\frac{T^{\omega-1/2}}{\Delta}\frac{\partial u^\alpha}{\partial \zeta} \quad (\alpha = 1, 2),$$

$$\beta_3\varphi^1 \frac{d\Delta}{d\xi} + \Delta\left(\beta_1\varphi^1 + \beta_2\varphi^2 + \beta_3\frac{\partial \varphi^1}{\partial \xi}\right) = 0, \quad G = Q^T \frac{T^{\omega-1/2}}{\Delta}\frac{\partial G}{\partial \zeta},$$

$$Q = \frac{2 - \theta}{\theta}\sqrt{\frac{\gamma\pi}{\gamma - 1}}\frac{1}{\mathrm{Re}}, \quad Q^T = \frac{(2 - \alpha)}{\alpha}\frac{2\gamma}{(\gamma + 1)}\sqrt{\frac{\gamma\pi}{\gamma - 1}}\frac{1}{\mathrm{Re\,Pr}},$$

where $\theta$ is the coefficient of diffuse reflection; $\alpha$ is the accommodation coefficient (in the calculations, we took $\theta = 1$, $\alpha = 1$).

We use the generalized Rankine–Hugoniot relations

$$\zeta = 1\colon \; u^\alpha = 1 - \frac{\mu\rho\sqrt{g}}{\mathrm{Re}\Delta}\frac{\partial u^\alpha}{\partial \zeta} \quad (\alpha = 1, 2),$$

$$G = 1 - \frac{\mu\rho\sqrt{g}}{\mathrm{Re\,Pr}\,\Delta}\frac{\partial}{\partial \zeta}\left[G - \frac{1 - \mathrm{Pr}}{1 - G_w}\beta_0 (u^1)^2\right],$$

$$p = \frac{1}{g}, \quad p_1 = -2\beta_1, \quad p_2 = -\frac{2}{g}\left(\beta_2 + \frac{\beta_7}{\beta_2}\right),$$

$$\beta_3\varphi^1 \frac{d\Delta}{d\xi} + \Delta\left(\beta_1\varphi^1 + \beta_2\varphi^2 + \beta_3\frac{\partial \varphi^1}{\partial \xi}\right) = \frac{1}{\sqrt{g}}$$

for the shock wave. The friction coefficients in the neighborhood of the symmetry plane are calculated from the formulas

$$c_f^\alpha = \frac{\mu_* \partial (V_\infty u^\alpha)/\partial \xi_*^3}{\rho_\infty V_\infty^2/2} = \frac{2(\mu\rho)_w}{\mathrm{Re}\,\Delta}\left(\frac{\partial u^\alpha}{\partial \zeta}\right)_w \quad (\alpha = 1, 2).$$

The flow of heat to the surface of a body in a flow with slip is due to both conduction and friction and consists of two parts: $q = \lambda_* \partial T_* / \partial \xi_*^3 + \mu_* u_*^1 \partial u_*^1 / \partial \xi_*^3$ (the asterisk denotes dimensional quantities). The Stanton number $c_H$ is found from the formula

$$c_H = \frac{q}{\rho_\infty V_\infty (H_\infty - H_w)} = \frac{(\mu\rho)_w}{\mathrm{Re}\,\mathrm{Pr}\Delta}\left(\frac{\partial G}{\partial \zeta} - \frac{1-\mathrm{Pr}}{1-G_w}\beta_0 u^1 \frac{\partial u^1}{\partial \zeta}\right).$$

2. Equations (1.1) will be solved by the method of successive approximations. Here, the procedures employed are similar to those used in [9, 10]. The method was first proposed for two-dimensional boundary-layer problems in [11]. The authors of [9, 10] then developed a technique of successive approximation to solve two-dimensional problems of the theory of a viscous shock layer.

The momentum and energy equations are integrated twice over the coordinate $\zeta$ (from $\zeta$ to 1 and from 0 to $\zeta$). To solve the resulting system of integrodifferential equations, we construct an iterative process in which each successive approximation for the sought functions is expressed through the previous approximation so that all of the approximations satisfy the boundary conditions both for the body and for the shock wave. The iterative algorithm makes it possible to determine any number of approximations for the sought functions if the zeroth approximation is somehow given. The algorithm is similar in form to that presented in [10] and is thus not described here.

In the first approximation of this method, we obtained an analytic solution for pressure, the velocity components, the friction coefficients, and the Stanton number:

$$p_w = \frac{1-\beta_* t}{g}, \quad \beta_* = \frac{g\beta_1\beta_0 a}{3r\beta}, \quad t = 1 + 3b + 3b^2; \tag{2.1}$$

$$u^\alpha = \alpha_0 \eta^\alpha \Delta_\alpha \sqrt{g} + \alpha_1 \Delta_\alpha^2 (T^1(\zeta) - F^\alpha(\zeta)) \quad (\alpha = 1, 2), \tag{2.2}$$

$$G = \alpha_0 \eta \Delta_H \sqrt{g} + \alpha_1 \mathrm{Pr}\,\Delta_H^2 T^2(\zeta) + \frac{1-\mathrm{Pr}}{1-G_w}\beta_0 (u^1)^2;$$

$$c_f^\alpha = 2\eta^\alpha \Delta_\alpha / \sqrt{g}, \quad \eta^\alpha = r - at/3 - R^\alpha \quad (\alpha = 1, 2); \tag{2.3}$$

$$c_H = \eta \Delta_H / \sqrt{g}, \quad \eta = r - c(1 + (3/2)(b+d) + 3bd)/3,$$

$$a_1^\alpha = r(1 - a(1+b)) + \sqrt{g}\,\alpha_0 \eta^\alpha, \quad a_2^\alpha = \alpha_1 (T^1(1) - F^\alpha(1)),$$

$$b_1 = r(1 - c(1+d)) + \sqrt{g}\,\alpha_0 \eta, \quad b_2 = \alpha_1 \mathrm{Pr}\, T^2(1),$$

$$b_3 = 1 - \frac{1-\mathrm{Pr}}{1-G_w}\beta_0 (u_s^1)^2, \quad r = 1/2 + b,$$

$$\alpha_0 = \frac{((1/2)\pi\varepsilon(cd + G_w))^{1/2}}{g p_w}, \quad \alpha_1 = \frac{\mathrm{Re}\,\varepsilon(c(1-G_w))^{1/2}}{a\beta},$$

$$\Delta_\alpha = \frac{-a_1^\alpha + \sqrt{(a_1^\alpha)^2 + 4a_2^\alpha}}{2a_2^\alpha}, \quad \Delta_H = \frac{-b_1 + \sqrt{b_1^2 + 4b_2 b_3}}{2b_2},$$

$$T^\gamma(\zeta) = \sum_{i=-1}^{4}\sum_{j=0}^{3} \frac{l_i \tau_j^\gamma}{i+j+3/2}\left(n^{i+j+3/2} - d^{i+j+3/2}\right) \quad (\gamma = 1, 2, 3), \quad n = \zeta + d,$$

$$\tau_0^1 = r - at/3, \quad \tau_1^1 = ad(b - d/2), \quad \tau_2^1 = -aq/2, \quad \tau_3^1 = -a/6,$$

$$\tau_0^2 = \eta, \quad \tau_1^2 = dc(b - d/2), \quad \tau_2^2 = -cq/2, \quad \tau_3^2 = -c/6,$$

$$R^\alpha = \frac{S^\alpha}{3\beta} at + \frac{\varepsilon}{a\beta}\sum_{i=1}^{3}\sum_{j=0}^{3}\frac{q_i h_j^\alpha}{i+j}\left((1+d)^{i+j} - d^{i+j}\right), \quad q = b - d,$$

$$q_1 = G_w, \quad q_2 = c(1 - G_w) + 2gq_3, \quad q_3 = -\beta_0 a^2,$$

$$S^1 = \beta_1, \quad S^2 = \beta - g\beta_1 + \beta_7/\beta_2,$$

$$h_0^\alpha = -\frac{2}{a} - th_3^\alpha, \quad h_1^\alpha = 3q^2 h_3^\alpha, \quad h_2^\alpha = 3qh_3^\alpha,$$

$$h_3^\alpha = \frac{g}{3r\beta}(2\beta_1\beta^2 + k^\alpha), \quad k^1 = \beta_5/g, \quad k^2 = 2\beta_2^2 + \beta_6 + 4\beta_7 - \beta_0\beta_5,$$

$$F^\alpha(\zeta) = \frac{S_a^\alpha}{3\beta} T^3(\zeta) +$$

$$
+\frac{\varepsilon}{a\beta}\sum_{k=-1}^{4}\sum_{i=1}^{3}\sum_{j=0}^{3}\frac{l_k q_i h_j^{\alpha}}{i+j}\left((1+d)^{i+j}\frac{n^{k'}-d^{k'}}{k'}-\frac{n^{i+j+k'}-d^{i+j+k'}}{i+j+k'}\right), \tag{2.4}
$$

$$
k'=k+3/2,\ \tau_0^3=t,\quad \tau_1^3=-3q^2,\quad \tau_2^3=-3q,\quad \tau_3^3=-1,
$$

$$
p_0=1+\beta_* t,\quad l_{-1}=\frac{G_w p_0}{2c},\quad l_0=\left(1+\frac{qq_3}{c}\right)p_0,
$$

$$
l_1=\frac{q_3 p_0}{2c},\quad l_2=-\beta_*\left(3q+\frac{G_w}{2c}\right),\quad l_3=-\beta_*\left(1+\frac{5qq_3}{2c}\right),
$$

$$
l_4=-\frac{\beta_* q_3}{2c},\quad b=\frac{Q_* a}{1-2Q_* a},\quad d=\frac{Q_*^T a}{1-2Q_* a},\quad c=\frac{\mathrm{Pr}\,a}{1+\mathrm{Pr}\,a(1+d)-a(1+b)};
$$

$$
\mathrm{Pr}\,(1+a(Q_*^T-2Q_*))(a^4-E\varphi^2)+a^3\varphi=0,\quad \varphi=1-a(1+2Q_*)+Q_* a^2,
$$

$$
Q_*=\frac{\beta\sqrt{g}\,Q}{2},\quad Q_*^T=\frac{\beta\sqrt{g}\,Q^T}{2},\quad E=(1-G_w)\left(\frac{2\,\mathrm{Re}\,\varepsilon}{\beta}\right)^2,\quad \beta=\beta_1+\beta_2.
$$

Calculations performed with Eq. (2.4) and numerical calculations performed by the finite-difference method for bodies of different form showed that at Re $\gtrsim$ 5 the distribution along the surface of the heat flux referred to the heat flux at the critical point is only slightly affected by allowing for slip and the temperature jump on the surface. The above equations can thus be simplified, and we obtain the following simple formula for relative heat flux

$$
\frac{q}{q_0}=\frac{H}{gH_0}\frac{(1-(2/3)c)}{(1-(2/3)c_0)}\frac{T_0}{T}\frac{c-1+\sqrt{(1-c)^2+8\,\mathrm{Re}\sqrt{g}\,T/H}}{c_0-1+\sqrt{(1-c_0)^2+8\,\mathrm{Re}\,T_0/H_0}}, \tag{2.5}
$$

$$
T=\varepsilon\,\mathrm{Pr}\,(c(1-G_w))^{1/2}\left[\frac{1}{3}\left(1-\frac{7}{9}c\right)+\frac{G_w}{2c}\left(1-\frac{5}{7}c\right)\right],
$$

$$
c=\frac{a\,\mathrm{Pr}}{1-a(1-\mathrm{Pr})},\quad a^4(1-\mathrm{Pr})-a^3+E^*(1-a)^2=0,
$$

$$
E^*=g(1-G_w)\,\mathrm{Pr}\left(\frac{\varepsilon\,\mathrm{Re}}{H}\right)^2,
$$

where H is the mean curvature of the surface at the given point [$H = (k_1 + k_2)/2$]; $k_1$ and $k_2$ are the principal curvatures of the surface at this point; the subscript 0 denotes values of the respective quantities at the stagnation point.

It follows from analysis of (2.5) that with moderate and large Reynolds numbers (Re $\gtrsim$ 100), relative heat flux ceases to depend on Re, and Eq. (2.5) takes the form

$$
\frac{q}{q_0}=\frac{1}{g^{3/4}}\sqrt{\frac{H}{H_0}},\quad H=\frac{1}{2g}\left(\frac{f_{11}''}{g}+f_{22}''\right). \tag{2.6}
$$

The practical absence of the dependence of the distribution of relative heat flux over the surface on Re (at Re $\gtrsim$ 100) was confirmed by numerical calculations.

3. To evaluate the accuracy of the formulas obtained here, we made a systematic comparison of the analytic solution with a numerical solution obtained using a difference scheme of fourth-order accuracy with respect to $\zeta$ and second-order accuracy with respect to $\xi$. The results of calculations performed with the approximate analytic formulas were compared with the results of the numerical calculations in the neighborhood of the plane of symmetry of elliptical paraboloids and hyperboloids in flows with angles of attack. The comparison was made for a broad range of variation of the governing parameters of the problem: Re changed from 1 to 1000; the angle of attack $\alpha$ changed from 0 to 45°; the temperature factor $G_w$ changed from 0 to 0.3; $\gamma$ changed from 1.1 to 1.4. Comparison of the analytic and numerical solutions showed the good accuracy of the formulas for pressure, the friction coefficients, and heat transfer throughout the investigated ranges of the parameters of the problem. This accuracy is evident from Figs. 1-3, for example. These figures show the characteristic distributions of these quantities along the surface of an elliptical paraboloid $z = (1/2)(x^2 + ky^2)$ ($k = 0.8$) in a flow with angles of attack of 0, 45; 15 and 30°, respectively. Also shown are the characteristic distributions of the quantities along the surface of a hyperboloid $z = (1 + x^2 + y^2)^{1/2} - 1$ in a flow with an angle of attack of $\alpha = 30°$ (curves 1' and 3' in Fig. 3). Lines 1-3 correspond to Re = 1, 10, and 100. The solid lines show the numerical solution, while the dashed lines show the results calculated from Eqs. (2.1), (2.3), and (2.4) for $p_w$, $c_f^1$, and $c_H$. Curves 1'-4' in Fig. 1 are for $\alpha = 0$. The results are presented for $\gamma = 1.4$, $G_w = 0.1$, and Pr = 0.71.

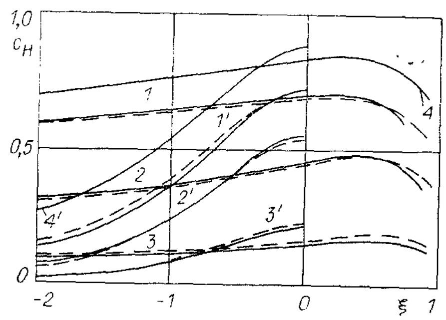


Fig. 1

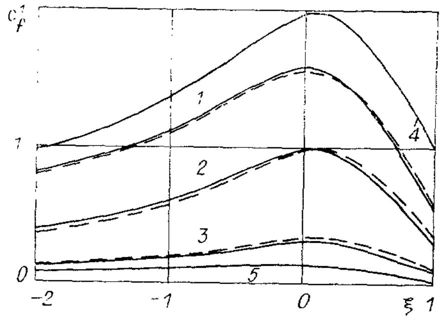


Fig. 2

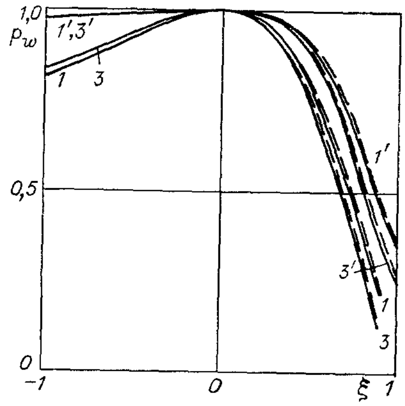


Fig. 3

Study of the dependence of the heat flux and shear stress on α showed that an increase in α is accompanied by a shift in these quantities from the critical point in the direction of a decrease in the radius of longitudinal curvature of the contour of the body. However, the absolute maximum of q and $c_f^1$ for the given body is reached at the critical point in a flow without an angle of attack. Thus, in the case of flow about an elliptical paraboloid (k = 0.8) with Re = 100, for α = 0, 15, 30, and 45° the maximum values of $c_H$ = 0.22, 0.21, 0.19, and 0.16.

An analysis of the results we obtained showed that the terms containing components of the pressure gradient that are tangential to the surface cannot be omitted from the momentum equations, since failure to account for them leads to a substantial error in the determination of the friction coefficients. This is demonstrated in Fig. 2, where lines 5 show data found with the same values of the parameters as for lines 3 – but without allowance for the terms containing the tangential components of the pressure gradient.

As in the case of axisymmetric flows, for small Re the effects of slip and the temperature jump on the surface depend appreciably on the distribution of the absolute values of shear stress and heat flux on the surface. This is shown in Figs. 1 and 2 (lines 4 were obtained with the same parameters as lines 1, but without allowance for slip). Here, the effect of slip is manifest to a greater extent in the case of flow about the body with an angle of attack on the side relative to the stagnation point where the radius of longitudinal curvature of the body's contour decreases.

We also checked the accuracy of simplified formula (2.5) for the heat-flux distribution based on the heat flux at the critical point. An example of a comparison of the analytic and numerical solutions is shown in Fig. 4 for an elliptical paraboloid (k = 0.8) in flows with angles of attack of 0 and 15° and Re = 10 and 100 (curves 1 and 2). The solid lines show the numerical solution, the dashed lines show the results calculated from Eq. (2.5), and the dot–dashed lines show the results calculated from Eq. (2.6). The results calculated from (2.6), being independent of Re, nearly (to within 1%) agree with the results calculated from (2.5) with Re = 100. Thus, at Re ≳ 100, Eq. (2.6) can be used to determine $q/q_0$.

The fact that the distribution of relative heat flux nearly ceases to be dependent on Re at Re ≳ 100 is illustrated in Fig. 5, which shows the distributions of $q/q_0$ and $c_f^1/c_{f0}^1$ along the line of flow on the elliptical paraboloid (k = 0.8, α = 30°) obtained from the

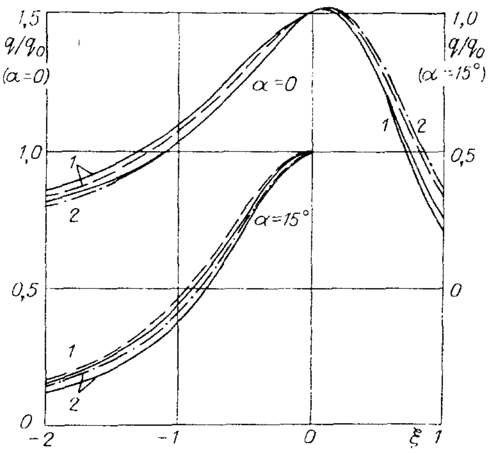


Fig. 4

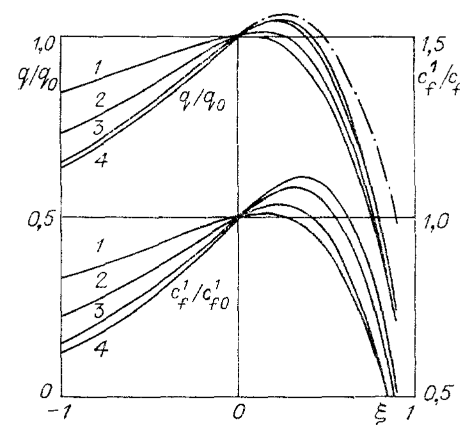


Fig. 5

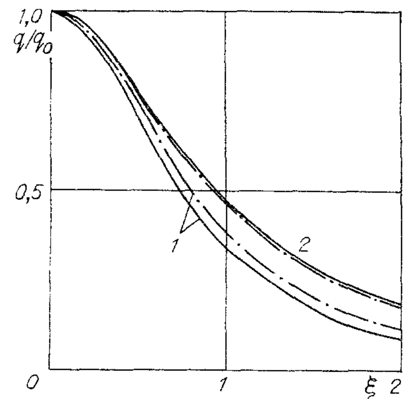


Fig. 6

numerical solution (solid lines) with Re = 1, 10, 100, and 700 (curves 1-4). It is evident that, in contrast to $q/q_0$, the distribution of $c_f{}^1/c_{f0}{}^1$ in the range $100 \leq Re \leq 700$ is still appreciably dependent on Re.

Comparison with the results of a numerical solution for different forms of bodies immersed in flows with different angles of attack show the high degree of accuracy of Eq. (2.6). This equation expresses the relative heat flux at a given point on the flow line as a function of the metric and the mean curvature of the surface at this point when $Re \gtrsim 100$. Examples of such a comparison for elliptical paraboloids with $k = 0.8$, $\alpha = 0$, 15°; $k = 0.8$, $\alpha = 30°$; $k = 0.25$ and 4 (curves 1 and 2) and $\alpha = 0$ are shown in Figs. 4-6, respectively; the results calculated with Eq. (2.6) (dot—dashed lines) agree well with the numerical solution of Eqs. (1.1) (solid lines).

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# INVERSE METHOD OF MEASURING ELECTRICAL CONDUCTIVITY IN A ROTATING MAGNETIC FIELD

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Among the noncontact methods available for measuring electrical conductivity [1], an important place is occupied by the method based on the determination of the torque acting on a specimen in a rotating magnetic field [2-5]. The specimen, in the form of a sphere or cylinder, is suspended on a thin elastic filament. The rotating magnetic field is created by means of two- or three-phase current or by the mechanical rotation of coils carrying a direct current. The torque acting on the specimen is determined from the angle of twist of the filament or by compensating for it. In the former case, a light source, mirror, and scale are needed. The electrical conductivity of the specimen is determined by an absolute or relative method. In the first case, use is made of the exact solution of the problem for a conducting sphere or cylinder in a rotating magnetic field [6].

The goal of the present study is to improve the method of measuring electrical conductivity in a rotating magnetic field with the specific aim of checking the purity of metals from their residual resistivity. The procedure for checking purity is based on the fact that at sufficiently low temperatures, the resistivity of metals is determined mainly by impurities and lattice defects. This technique is currently widely used. The purity of a metal is usually characterized by the ratio of its resistivities at room temperature and at the temperature of liquid helium. The main shortcoming of the method which involves the use of a rotating magnetic field is that the specimens must be small — usually about 1 cm. When it is necessary to study the distribution of impurities along long specimens, the only possible means is to cut them into small sections and perform separate measurements for each section. This significantly lengthens the time taken up by the measurement process and increases the consumption of liquid helium. Moreover, this method is more difficult still if the specimen has to be in a sealed ampul for the entire period of measurement.

To overcome these problems, we propose a different method: instead of the torque acting on the specimen, determine the torque acting on the coils which create the magnetic field. A lightweight platform with coils transmitting a two- or three-phase alternating current is suspended on elastic filaments. When the specimen is moved into the space between the coils, eddy currents develop in this space. The interaction of these currents with the rota-

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